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Inverse analysis of thermomechanical upsetting tests using gradient method with semi-analytical derivatives ☆

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Abstract

The starting point of this work is the need of precise and correct input data for material forming codes. The use of these codes as a direct model for inverse analysis of the processes permits to extend the validity range of the thermomechanical parameters in terms of temperature, strain and strain rate. The identification software was developed on the basis of the 2D and of a 3D finite element code (FORGE2® and FORGE3®) simulating forming processes and using a thermo–elasto–viscoplastic behaviour. The optimisation problem is based on a Gauss Newton algorithm and necessitates the evaluation of the derivatives of the cost function and of the sensitivity matrix to solve the system. Different methods are proposed to evaluate these derivatives. We have studied deeply analytical evaluation, finite difference techniques and recently semi-analytical derivatives. In this paper we present the main feature of the semi-analytical derivatives and the comparison with numerical ones on the parameter identification during upsetting tests. The semi-analytical method of sensitivity analysis for inverse problems is very attractive thanks to the compromise between computational time and ease of derivatives evaluation. Especially for parameter identification in material forming domain, this technique seems to be promising.

Keywords: Inverse analysis; Rheological and friction parameter identification; Semi-analytical derivatives

1. Introduction

At the present time, a large part of finite element codes are supposed to be sufficiently robust and stable to have confidence in their results. Ones can consequently question the validity of input data in case of bad results of the code. As an example, the results of a material forming simulation are very sensitive to the input data like rheological law and tribological model. The evaluation of these thermomechanical parameters is crucial. The parameter identification is a necessary step toward optimisation of forming process simulation. A parameter identification module, working as an inverse code, was developed. The direct model is based on a 2D and on a 3D finite element software FORGE2® and FORGE3®) devoted to the simulation of metal forming processes. The principle of identification is to determine the optimal set of thermomechanical parameters minimising the distance between experimental and numerical results. This deviation is expressed by a cost function using the least square method which has an interest for non-linear models [1]. An optimisation algorithm based on the Gauss Newton method is used to solve the problem. It requires experimental quantities like the load or the shape of the part, and it delivers various coefficients involved in the constitutive and friction laws. Sensitivity analysis of the software response with respect of the thermomechanical data is then absolutely necessary.

Several methods are used for sensitivity evaluation: finite difference methods [2], analytical techniques and semi-analytical evaluation. The analytical method is appreciated by its efficiency and its accuracy in large deformation with remeshing [3]. The advantage of the finite difference derivatives is the simplicity of their evaluation and implementation even for complex mathematical models [2,4]. Thanks to this technique, the inverse module can play the role of black box and be connected to any direct mode. Moreover, it can be quickly implemented. The main drawbacks, especially in material forming, are the sensitivity of these derivatives to the remeshing and the bad precision of the derivatives especially for elasto-viscoplastic behaviour. The third direction is the semi-analytical or quasi-analytical estimation of the sensitivity matrix. This method is studied mainly for structural

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Nomenclature

σ	stress tensor	kPa	
s	deviatoric part of the stress tensor	kPa	
p	hydrostatic pressure	kPa	
$\Omega(t)$	domain of the piece at time t		
$\partial\Omega_c$	contact surface		
$\partial\Omega_n$	free surface		
τ	friction shear stress	kPa	
ρc	heat capacity	$J{\cdot}K^{-1}{\cdot}m^{-3}$	
k	conductivity	$W{\cdot}K^{-1}{\cdot}m^{-1}$	
h	global heat transfer coefficient	$W{\cdot}K^{-1}{\cdot}m^{-2}$	
λ	set of thermo–mechanical parameters		
$Npar$	number of thermo–mechanical parameters		
R	mechanical residue		
$[C]$	heat capacity matrix		
$[K]$	heat conductivity matrix		
Q	load vector		
N	load	$kg{\cdot}m{\cdot}s^{-2}$	
G	geometry data	m	
Ψ	cost function		
J_λ	Gauss–Newton matrix		
S	sensitivity matrix		

optimisation [5,6]. It represents a good alternative between the finite difference method using the direct model and the analytical techniques, for which is necessary to go deeply in the core of the code to differentiate. Some authors exhibit limitations of the semi-analytical method mainly due to the imprecision and especially depending on the type of elements chosen: shell, beam, solid, membrane. The error increases with the number of elements chosen.

In this paper we use the semi-analytical method for parameter identification in material forming. The aim of this study is to perform a simultaneous rheological and tribological identification during classical upsetting tests. One of the main problems is determination of the friction coefficient. In fact, if the results of the tests are not sensitive enough with respect to the tribology, it will be extremely difficult to identify the friction parameter. Comparisons between finite difference method and semi-analytical derivatives for sensitivity evaluation are discussed, using 2D (FORGE2®) and 3D (FORGE3®) direct model.

2. Direct and inverse problem

2.1. Direct finite element model

The direct model (FORGE2® or FORGE3®), is a finite element software devoted to the simulation of material forming processes. The mechanical equilibrium is represented by the following equations:

$$\begin{cases} \operatorname{div} s - \nabla p = 0 & \text{in } \Omega(t) \\ \operatorname{div} v = 0 & \text{in } \Omega(t) \\ \left.\begin{array}{l} \sigma_n \leqslant 0 \\ (v - v_{\text{tool}}) \cdot n \leqslant 0 \\ \sigma_n (v - v_{\text{tool}}) \cdot n = 0 \end{array}\right\} & \text{on } \partial\Omega_c \\ \sigma n = 0 & \text{on } \partial\Omega_n \end{cases} \tag{1}$$

where s is the deviatoric part of the Cauchy stress tensor, p the hydrostatic pressure, $\Omega(t)$ the domain occupied by the workpiece and $\partial\Omega$ the boundary of the domain, v the velocity field, v_{tool} tools velocity, σn the normal stress vector and $\sigma_n = \sigma n \cdot n$.

The elastic part of strain tensor is expressed with the Hooke's law and the viscoplastic part with the Norton–Hoff's law. The material obeys to the Von Mises yield criterion. The material is assumed homogeneous and isotropic. The rheological and tribological models are represented by:

$$\begin{aligned} \sigma_0 &= K\left(T, \bar{\varepsilon}\right) \exp(\beta/T) \left(\bar{\varepsilon} + \bar{\varepsilon}_0\right)^{n(T,\bar{\varepsilon})} \left(\sqrt{3}\dot{\bar{\varepsilon}}\right)^{m(T,\bar{\varepsilon})} \\ \tau &= -\frac{\overline{m}\sigma_0}{\sqrt{3}} \frac{\left\| (v - v_{\text{Vtool}}) \cdot t \right\|}{(v - v_{\text{Vtool}}) \cdot t} \end{aligned} \tag{2}$$

with K the consistency of the material, n the strain hardening coefficient, m the strain rate sensitivity index, β thermal parameter, $\bar{\varepsilon}$ the equivalent plastic strain, $\dot{\bar{\varepsilon}}$ the equivalent plastic strain rate and $\dot{\varepsilon}$ the strain rate tensor, σ_0 the plastic yield stress, τ friction shear stress, $\overline{m}$ the friction coefficient (Tresca model), t the tangential vector. The input data of the model are the mathematical formulation of both the rheological and the tribological models represented by Eq. (2), together with the operating conditions in terms of temperature and tools velocities.

The thermal equilibrium is written as following:

$$\begin{cases} \rho c \dfrac{\mathrm{d}T}{\mathrm{d}t} - \nabla \cdot (k \nabla T) = \dot{W} & \text{in } \Omega(t) \\ -k \nabla T \cdot n = h(T_p - T_{\text{ext}}) & \text{on } \partial\Omega \end{cases} \tag{3}$$

with ρc the heat capacity, k the conductivity, $\dot{W} = \sigma : \dot{\varepsilon}$ the heat part dissipated through plastic deformation, T_p the part surface temperature, T_{ext} the surrounding temperature, h the global heat transfer coefficient and n the external surface normal. The thermal boundary conditions can be expressed as:

– radiation and convection with:

$$h = h_{\text{cv}} + h_r \quad \text{on } \partial\Omega_n$$
$$h_r = \varepsilon_r \sigma_r (T_p + T_{\text{ext}}) \left(T^2 + T_{\text{ext}}^2\right)$$

and h_{cv} depending on the material and on the external properties;

– diffusion condition on $\partial\Omega_c$ with h depending on the materials in contact.

The mechanical problem stated Eq. (1) is expressed using the principle of virtual work and solved with a Newton–Raphson algorithm. An updated Lagrangian method combined with a P2/P0 velocity/pressure mixed finite element formulation is used. For large deformations, a remeshing algorithm is automatically used. The discretised form of the principle of virtual work is written at each time step t:

$$R^t\left(\lambda, X^t, T^{t-\Delta t}, V^t\right) = 0 \tag{4}$$

Where λ is a set of rheological and tribological parameters representing the material model, X^t is the vector of coordinates of the mesh nodes, $T^{t-\Delta t}$, V^t, respectively, discretised temperature and velocity. As an example with the formulation given by Eq. (2) and the assumption of K, n and m constants:

$$\lambda = (K, \beta, n, m, \overline{m}) \tag{5}$$

The system represented by Eq. (4) is non-linear and solved by a Newton–Raphson method.

The thermal equilibrium is written in a variational form through Galerkin method. The finite element discretisation of the thermal equation is done using piecewise linear interpolation of temperature T (P1 elements). The discretised form of thermal problem is:

$$[C]\frac{\mathrm{d}T^t}{\mathrm{d}t} + [K]T^t = Q\left(V^t\right) \tag{6}$$

where $[C]$ is the capacity matrix and $[K]$ the conductivity matrix. Q is the loading vector.

Then solving the thermomechanical equations Eq. (4) and Eq. (6), the direct model provides the distribution, in time and space, of local variables like stress, strain, temperature, and also global variables like force F^{calc} and geometry G^{calc}. These terms depend on the input data chosen for the finite element simulation:

$$N^{\mathrm{calc}}(\lambda, V, X) = \left(F^{\mathrm{calc}}(\lambda, V, X), G^{\mathrm{calc}}(\lambda, V, X)\right) \tag{7}$$

These variables can be compared with those coming directly from the experimental set up denoted by N^{exp}. These quantities are called observables. The software, working as an inverse problem, determines the set of parameters λ minimizing the gap between experimental N^{exp} and calculated values N^{calc}.

2.2. Inverse model

The deviation between computed observables and experimental ones is expressed via an objective function Ψ written using the classical least square method. The optimisation problem is then:

$$\begin{cases} \mathrm{Min}_{\lambda\in C}\,\Psi(\lambda) \\ \Psi\left(\lambda, N^{\mathrm{calc}}(\lambda), N^{\mathrm{exp}}\right) \\ \quad = \sum_{l=1}^{n} \dfrac{(N_l^{\mathrm{calc}}(\lambda) - N_1^{\mathrm{exp}})^2}{\beta_l} & \text{with } \beta_l = \left(N_l^{\mathrm{exp}}\right)^2 \quad \text{or} \\ \quad = \dfrac{\sum_{l=1}^{n}(N_l^{\mathrm{calc}}(\lambda) - N_l^{\mathrm{exp}})^2}{\beta} & \text{with } \beta = \sum_{l=1}^{n}\left(N_l^{\mathrm{exp}}\right)^2 \end{cases} \tag{8}$$

Where n is the number of experimental points, β is the weight function associated to the experimental points and C is the parameter constraint space. Depending on the quality of the experimental information, the weight function could be constant for all the n-points or varying from one point to another.

A Gauss–Newton method is used to solve the problem:

$$\lambda^k = \lambda^{k-1} - \left[J_\lambda\left(\lambda^{k-1}\right)\right]^{-1}\frac{\mathrm{d}\Psi}{\mathrm{d}\lambda}\left(\lambda^{k-1}\right) \tag{9}$$

with:

$$\begin{cases} \dfrac{\mathrm{d}\Psi}{\mathrm{d}\lambda} = 2\sum_{l=1}^{n}\beta_l\left(N_l^{\mathrm{calc}}(\lambda) - N_l^{\mathrm{exp}}\right)\dfrac{\mathrm{d}N_l^{\mathrm{calc}}}{\mathrm{d}\lambda} \\ J_\lambda \cong 2\sum_{l=1}^{n}\beta_l\dfrac{\mathrm{d}N_l^{\mathrm{calc}}}{\mathrm{d}\lambda}\left[\dfrac{\mathrm{d}N_l^{\mathrm{calc}}}{\mathrm{d}\lambda}\right]^{\mathrm{T}} \end{cases} \tag{10}$$

J_λ is the Jacobian matrix of $\frac{\mathrm{d}\Psi}{\mathrm{d}\lambda}$ with respect to λ, neglecting the second order derivatives. The convergence of algorithm (8) depends on the influence of the parameters λ on the observable N and thus on the sensitivity matrix $\mathrm{S} = \frac{\mathrm{d}N}{\mathrm{d}\lambda}\cdot S$ can be evaluated either by a finite difference method or by analytical differentiation [2,3]. The disadvantage of the first one is that the computational time increases significantly with the number of parameters. In this paper we propose to compare two ways for sensitivity matrix evaluation: finite difference method and semi-analytical derivatives.

Of course, the uniqueness of the solution is difficult to prove as problem (8) is highly non-linear, then we can often fall into local minima. Nevertheless, mechanical parameter identification is favourable for using such techniques in which, in general case, we have a good idea about the variation range. Then the initial guess could be easily chosen. Secondly, the ratio between number of experimental points (more than one thousand) and the number of parameters to be identified (around 20) vary also in a positive way.

3. Sensitivity analysis

The sensitivity matrix evaluation using finite difference method is based on the first order approximation of S: for each parameter λ_j $(j = 1, NPar)$, the derivatives are:

$$\begin{aligned} S_{ij} &= \frac{\mathrm{d}N_i^{\mathrm{calc}}(\lambda)}{\mathrm{d}\lambda_j} \\ &= \frac{N_i^{\mathrm{calc}}(\lambda + \Delta\lambda_j e_j) - N_i^{\mathrm{calc}}(\lambda)}{\Delta\lambda_j} + \mathrm{O}(\Delta\lambda_j) \end{aligned} \tag{11}$$

e_j: jth vector of the canonical basis of IR^{Npar}.
Npar: number of parameters to identify.

This method is very easy to implement as it is external to the direct model and any rheological or tribological model can be differentiated. Nevertheless the main drawback are the high CPU time for high number of parameters (it necessitates $(NPar + 1)$ simulations), and their sensitivity to remeshing. Semi-analytical differentiation method aims to avoid these problems. The method proposed in this paper takes advantages of both the numerical differentiation and the analytical evaluation of the derivatives. This formulation is independent of the remeshing algorithm (no discontinuity of the computed derivatives when remeshing), and we can differentiate any kind of mathematical model without additional effort. The sensitivity matrix evaluation is based upon the direct differentiation of the discrete problem represented by Eq. (4). The sensitivity matrix is written:

$$S = \frac{\mathrm{d}N}{\mathrm{d}\lambda} = \lim_{\theta\to 0} \frac{N(\lambda^\theta, X^\theta, \mathrm{V}^\theta) - N(\lambda, X^t, V^t)}{\theta\lambda} \tag{12}$$

With:

$$\begin{cases} \lambda^\theta = \lambda + \theta\lambda \\ X^\theta = X^t + \dfrac{\mathrm{d}X^t}{\mathrm{d}\lambda}\theta\lambda \end{cases} \tag{13}$$

Where V^θ is defined by:

$$V^\theta = V^t + \frac{\mathrm{d}V^t}{\mathrm{d}\lambda}\theta\lambda \tag{14}$$

$\frac{\mathrm{d}V^t}{\mathrm{d}\lambda}$ is obtained thanks to the differentiation of the direct problem (4):

$$\frac{\mathrm{d}R^t}{\mathrm{d}\lambda} = \left.\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right|_v + \frac{\partial R}{\partial V}\frac{\mathrm{d}V^t}{\mathrm{d}\lambda} = 0 \tag{15}$$

It comes:

$$\frac{\mathrm{d}V^t}{\mathrm{d}\lambda} = -\left(\frac{\partial R}{\partial V}\right)^{-1}\left.\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right|_v = -H^{-1}\left(V^t\right)\left.\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right|_v \tag{16}$$

The inverse of the Hessian matrix (denoted $H^{-1}(V^t)$) is already evaluated during the resolution of the direct problem Eq. (4) through the Newton–Raphson algorithm. The partial derivatives of R with respect to λ keeping V^t constant in Eq. (16) could be analytically calculated or evaluated by a finite difference method. For an elastic-plastic model using Prandtl–Reuss equations, the analytical derivatives are long and difficult to obtain. Then, a finite difference scheme is chosen. All the quantities are then simultaneously perturbed (except the velocity) for a forward finite difference method:

$$\left.\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right|_v = \lim_{\theta\to 0} \frac{R(\lambda^\theta, X^\theta, V^t, T^\theta) - R(\lambda, X^t, V^t, T^{t-\Delta t})}{\theta\lambda} \tag{17}$$

where $T^\theta = T^{t-\Delta t} + \frac{\mathrm{d}T^{t-\Delta t}}{\mathrm{d}\lambda}\theta\lambda$ $\frac{\mathrm{d}T^{t-\Delta t}}{\mathrm{d}\lambda}$ has ever been computed at the last increment (cf. Eq. (19)). Thanks Eq. (4) we obtain:

$$\left.\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right|_v = \lim_{\theta\to 0} \frac{R(\lambda^\theta, X^\theta, V^t, T^\theta)}{\theta\lambda} \tag{18}$$

As the direct model, the inverse model requires the resolution of a thermo–mecanical coupled problem. Once $\frac{\mathrm{d}V^t}{\mathrm{d}\lambda}$ is estimated, it is necessary to compute $\frac{\mathrm{d}T^t}{\mathrm{d}\lambda}$ from the differentiation of Eq. (6), using a semi-analytical scheme.

$$[C]\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}T^t}{\mathrm{d}\lambda}\right) + [K]\frac{\mathrm{d}T^t}{\mathrm{d}\lambda} = \frac{Q(V^\theta) - Q(V^t)}{\theta\lambda} \tag{19}$$

After remeshing, the nodal values of the derivatives of X^t and T^t are kept using an accurate "transport". In order to determine the friction coefficient, the simulated tests should be sensitive enough with respect to friction. In the opposite case, identification of friction coefficient should not be possible. However, an example will be given in Section 4, showing that it is possible to quantify the friction with an upsetting test for which the cylinder is long enough. Many simulation tests have been suggested. In case of forging, ring test and cylinder upsetting are well-known. The aim of this study is to perform a tribological identification thanks to the influence of friction on the shape during upsetting. The software was improved with the use of a new observable: the barreling $G = (\Delta R)$ for a cylinder upsetting (Fig. 1). During cylinder upsetting, the force is influenced by the velocity of the punch and not much by the friction coefficient, whereas the barreling of the cylinder is sensitive to the friction coefficient values but not to the tool velocity. With this remark, rheology and tribology could be identified separately. The force allows determination of the rheological parameters for any kind of friction conditions at the interface and the geometry allows evaluation of the friction coefficient with material parameters provided by the first identification.

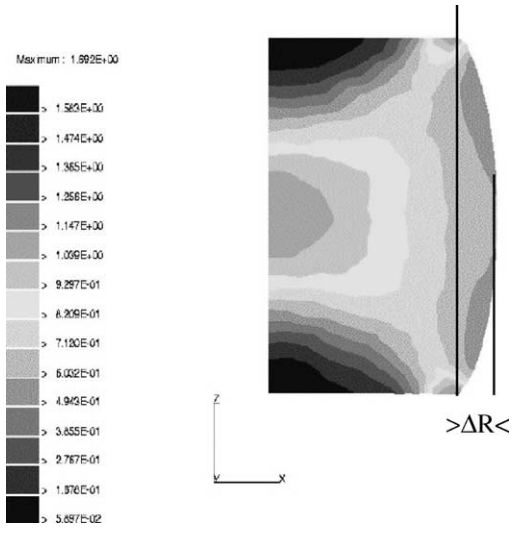

Fig. 1. Barrelling ΔR of upsetting test (half part simulated) using FORGE2®.

4. Results obtained by a numerical identification

4.1. Finite difference evaluation of sensitivity matrix

A numerical identification is made with an artificial experimental space: after several numerical simulations using the direct model FORGE2®, an artificial space supplying (F, G) is created. Then, the identification software should supply the values of the rheological and tribological parameters used to create this space. Such values are called nominal values. The interest of this approach is to validate the software numerically and to prove the efficiency of the geometry for the evaluation of friction values. A cylinder upsetting was carried out with two geometries (Height/Diameter = 0.75, and Height/Diameter = 1.5), currently used in experiment. The behavior is elasto-viscoplastic with a Tresca model (Eq. (2)) at the interface. The elastic parameters are $E = 200\,000$ Mpa, $\nu = 0.34$ and the nominal values are $K_0 = 50.2$, $m = 0.14$, $n = 0.07$, $\overline{m} = 0.8$. To start identification, the initial values of the coefficients are $K_0 = 10$, $m = 0.6$, $n = 1$ and $\overline{m} = 0.2$. The identified parameters are in the main cases obtained when the condition $\Psi \leqslant 10^{-4}$ is satisfied. Because of the sensitivity with respect to the strain rate, three upsetting tool velocities are used for a complete identification: 3.75 mm·s^{-1}, 7.5 mm·s^{-1} and 15 mm·s^{-1}. A 50% upsetting is made.

Authors noticed that the information on geometry evolution is always needed for a low friction. For a high friction and $H/D = 0.75$, the force allows convergence, which is not the case for the cylinder of ratio $H/D = 1.5$ (Tables 1 and 2). For the scenario illustrated in Table 1 and for $H/D = 0.75$, the influence of the friction on the geometry (G) during identification, while the force (F) is not much affected (5 step: $\overline{m} = 0.2$, 0.733, 0.804; 0.799, 0.8). However, an identification on $\overline{m} = 0.8$ with this ratio and the force only was possible (Table 1). For the complete identification with a ratio of $H/D = 1.5$, the stop condition is not satisfied with (F, G). In fact, the lack of remeshing does not allow a convergence with a good accuracy.

Table 1
Results of an identification on $\overline{m}$ with or without the help of the geometry

Observable	F $H/D = 0.75$	(F, G) $H/D = 0.75$	F $H/D = 1.5$	(F, G) $H/D = 1.5$
Optim. step	3	4	3	5
$\overline{m}$	0.801117	0.800050	0.200626	0.799848
Error	6.65×10^{-5}	2.95×10^{-6}	No conv.	1.26×10^{-5}

Table 2
Results of a complete identification with or without the help of the geometry

Observable	F $H/D = 0.75$	(F, G) $H/D = 0.75$	F $H/D = 1.5$	(F, G) $H/D = 1.5$
Optim. step	10	5	11	6
K	50.2067	50.2013	50.8986	50.0325
m	0.139999	0.140005	0.139850	0.139098
n	0.070012	0.070019	0.061681	0.066750
$\overline{m}$	0.798727	0.800073	0.010225	0.694427
Error	4.35×10^{-5}	3.20×10^{-5}	No conv.	9.40×10^{-3}

4.2. Semi-analytical evaluation of sensitivity matrix

4.2.1. Evaluation with the 2D software

Comparisons were made between semi-analytical and numerical derivatives. As an example, for 3 parameters, K_0, n and m the finite difference method, used in Section 4.1, requires 6 finite element simulations per experimental curve and per optimisation iteration. The semi-analytical method requires only 3 simulations per iteration and per experimental curve. This leads to faster and more precise identification with the semi-analytical derivatives. In this part we focus only on the rheological parameter identification with the help of the compression force knowledge. The nominal values of thermomechanical parameters are the same as the previous ones. A cylinder upsetting was carried out with

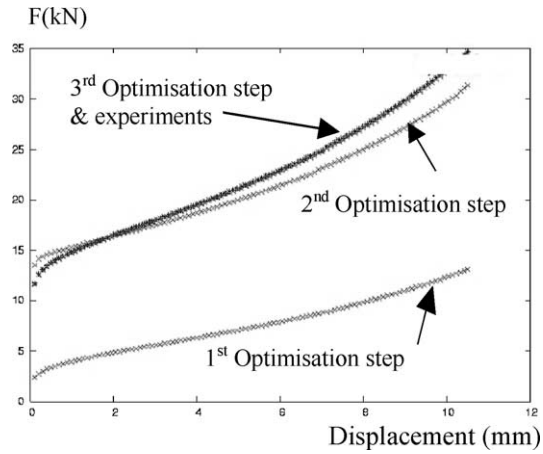

Fig. 2. Computed and Experimental compression forces-displacement curves for three optimisation iterations, without remeshing and with finite difference derivatives.

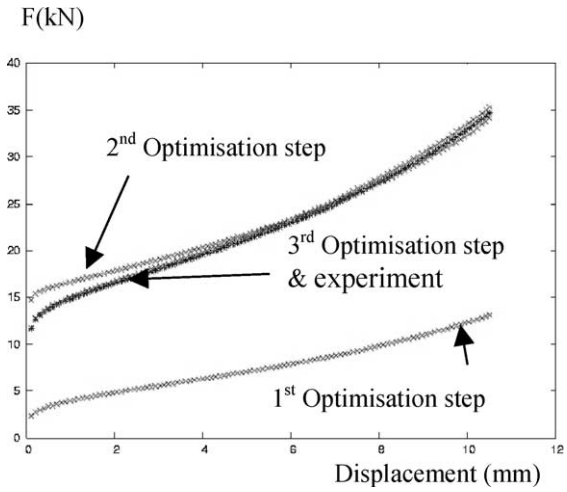

Fig. 3. Computed and Experimental compression forces-displacement curves for three optimisation iterations, without remeshing and with semi-analytical derivatives.

a ratio of $H/D = 1.5$ currently used in experiment. Usually this geometry is more sensitive to the rheology than the shorter one where the friction evolution at the interface plays a non-negligible role in the material flow. Figs. 2 and 3 show the evolution of the compression force during upsetting test versus the punch displacement without remeshing during the process simulation. In Fig. 2 the sensitivity matrix was based on finite difference technique while in Fig. 3 it was based on semi-analytical evaluation. Both techniques converge to the nominal values with smaller CPU time for the semi-analytical evaluation. The forces evaluated by the software during optimisation step are compared to the experimental ones.

The discrepancy with the experimental results is more important in Fig. 3. In Figs. 4 and 5, the same identification but with remeshing during the simulation are plotted. In Fig. 4, the computation failed with numerical derivatives, then only the 1st optimisation step is represented. In Fig. 5, it is not possible to distinguish the experimental result from the 2nd and final optimisation iteration.

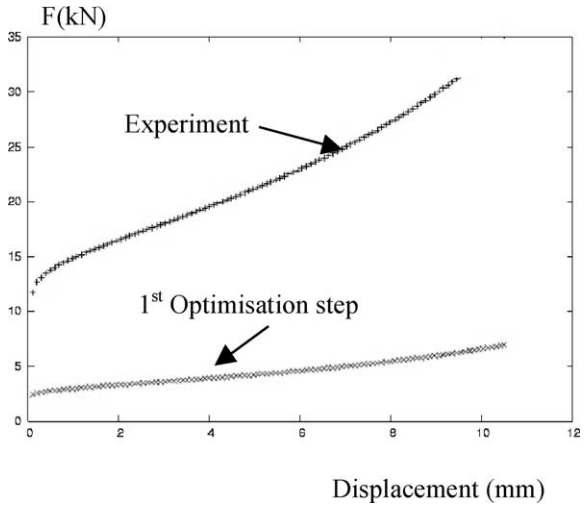

Fig. 4. Comparison between Computed and Experimental compression forces-displacement curves for one optimisation iterations (no convergence), with remeshing and with finite difference derivatives.

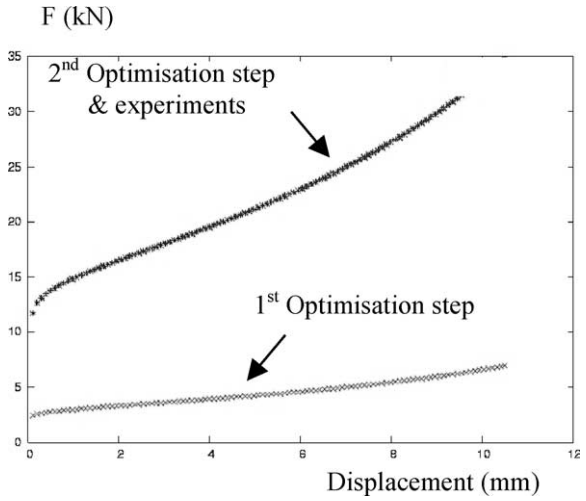

Fig. 5. Computed and Experimental compression forces-displacement curves for two optimisation iterations, with remeshing and with semi-analytical finite derivatives.

4.2.2. Evaluation with the 3D software

We consider the same test case. The identification is done using a 3D direct model (FORGE3®). The formulation of the mechanical and of the thermal problem is similar to FORGE2®. The use of a 3D direct model should allow for the identification of mechanical parameters from 3D rheological tests [7] (Plane strain compression test for example). We consider the same geometry as in the previous examples (Fig. 6) and we compare the efficiency of the finite difference method and of the semi-analytical derivatives using remeshing.

The computations with remeshing have been done for a 50% upsetting. The finite difference method does not converge in this case. This is probably due to the diffusion

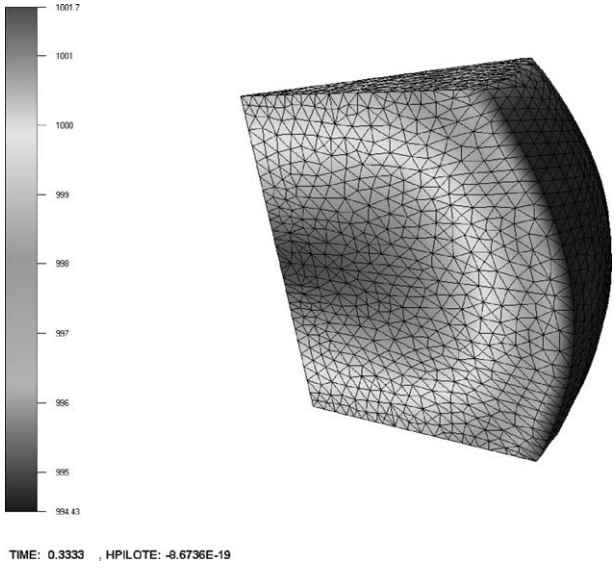

Fig. 6. Simulation with FORGE3® of the uniaxial compression of a section of a cylinder.

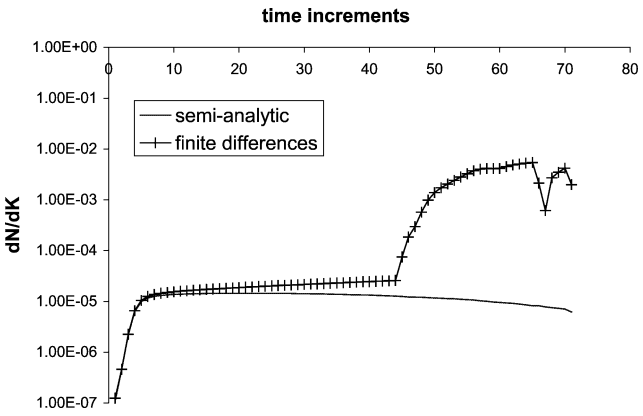

Fig. 7. Evolution of $\mathrm{d}N/\mathrm{d}K$ with respect to the iterations of Gauss–Newton algorithm with remeshing.

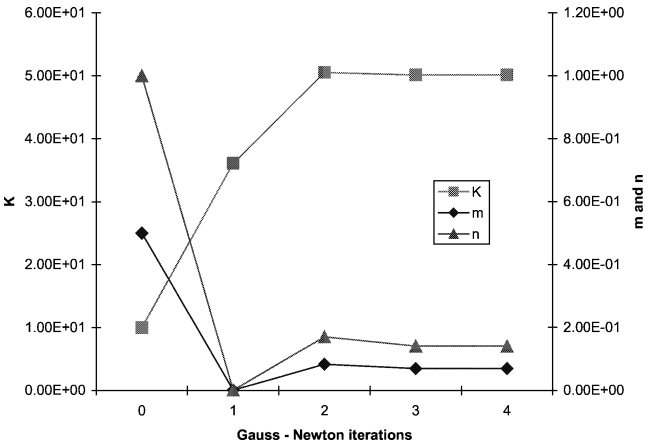

Fig. 8. Evolution of the parameters with respect to the iterations of Gauss–Newton algorithm with remeshing.

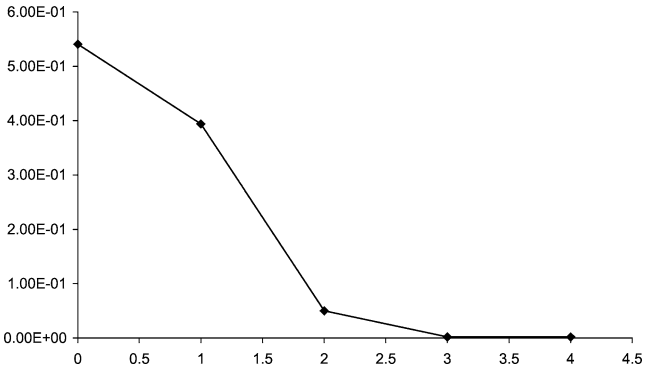

Fig. 9. Evolution of the cost function with respect to the iterations of Gauss–Newton algorithm with remeshing.

error caused by transfer of history variables after remeshing. For example, we can see a jump in the curve representing the derivatives of the load (N) with respect to the consistency K, versus the displacement (Fig. 7). This jump is not present when using the semi-analytical derivatives (Eq. (12)), as shown in Fig. 6. Sensitivity to remeshing seems to be more important with the 3D direct model. Then, Gauss–Newton algorithm converges with remeshing only using the semi-analytical method (Figs. 8 and 9). This method seems to be very precise and the convergence is reached after 4 Gauss–Newton iterations. It underlines that finite difference method cannot be used in such a case.

5. Conclusion

The use of semi-analytical derivatives in sensitivity analysis is a good alternative to the finite difference method or analytical evaluation. Especially when the complexity of material model is increasing as the elasto-viscoplastic behaviour and sophisticated friction law. It is also well known that numerical derivatives are highly mesh-dependent. The influence of remeshing and, above all, transport of variables from the old mesh to the new one is deterministic for the choice of precise evaluation of sensitivity matrix. The semi-analytical differentiation scheme, with the 2D and the 3D formulation, is stable, contrary to the finite difference method.

The future work will be the use of this technique for identification of material parameters including heat transfer coefficients on real experimental observation during forming processes like inertia friction welding, forging and hydroforming.

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